2018-2019 Guide

April 29- May 24th

Eureka

Module 6: *Foundations of Multiplication and Division*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Table of Contents

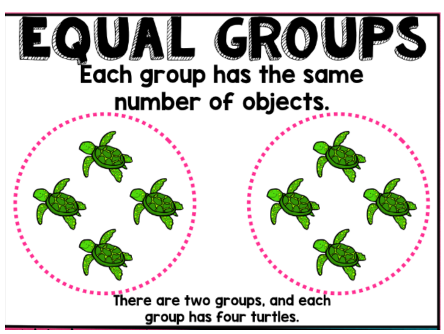
Module 6 Performance Overview

- In Topic A, students begin by making equal groups using concrete materials, learning to manipulate a given number of objects to create equal groups, and progress to pictorial representations where they may begin by circling a group of 5 stars, adding 5 more, and then adding 5 more. They determine the total and relate their drawings to the corresponding repeated addition equation. Students calculate the repeated addition sums by adding on to the previous addends, step-by-step, or by grouping the addends into pairs and adding. By the end of Topic A, students draw abstract tape diagrams to represent the total and to show the number in each group as a new unit. Hence, they begin their experience toward understanding that any unit may be counted. .This is the bridge between Grades 2 and 3. Grade 2 focuses on the manipulation of place value units, whereas Grade 3 focuses on the manipulation of numbers 1 through 10 as units.

- In Topic B, students organize the equal groups created in Topic A into arrays, wherein either a row or column is seen as the new unit being counted. They use manipulatives to compose up to 5 by 5 arrays one row or one column at a time and express the total via repeated addition equations. As Topic B progresses, students move to the pictorial level to represent arrays and to distinguish rows from columns by separating equal groups horizontally and vertically. .Then, they use same size square tiles, moving them closer together in preparation for composing rectangles

- In Topic C. Topic B concludes with students using tape diagrams to represent array situations and the RDW process to solve word problems. In Topic C, students build upon their work with arrays to develop the spatial reasoning skills they need in preparation for Grade 3's area content. They use same-size squares to tile a rectangle with no gaps or overlaps and then count to find the total number of squares that make up the rectangle.

- Topic D focuses on doubles and even numbers, thus setting the stage for the multiplication table of two in Grade 3



Module 6: Foundations of Multiplication and Division ***Pacing:*** **24 Days**		
Topic	**Lesson**	**Student Lesson Objective/ Supportive Videos**
Topic A: Formation of Equal Groups	Lesson 1&2	Use manipulatives to create equal groups. Use math drawings to represent equal groups, and relate to repeated addition https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 4	Represent equal groups with tape diagrams, and relate to repeated addition https://www.youtube.com/watch?v
Topic B: Arrays and Equal Groups	Lesson 5	Compose arrays from rows and columns and count to find the total using objects. https://www.youtube.com/watch?v
	Lesson 6	Decompose arrays into rows and columns, and relate to repeated Addition https://www.youtube.com/watch?v
	Lesson 7	Represent arrays and distinguish rows and columns using math Drawings https://www.youtube.com/watch?v
	Lesson 9	Solve word problems involving addition of equal groups in rows and columns https://www.youtube.com/watch?v
Mid-Module Assessment Task		
Topic C: Rectangular Arrays as a Foundation For Multiplica-	Lesson 10&11	Use square tiles to compose a rectangle and relate to the array model. https://www.youtube.com/watch?v https://www.youtube.com/watch?
	Lesson 12	Use math drawings to compose a rectangle with square tiles https://www.youtube.com/watch?v
	Lesson 13	Use square tiles to decompose a rectangle https://www.youtube.com/watch?v

tion and Division		
	Lesson 14	Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares https://www.youtube.com/watch?v
	Lesson 15	Use math drawings to partition a rectangle with square tiles, and relate to repeated addition https://www.youtube.com/watch?v
	Lesson 16	Use grid paper to create designs to develop spatial structuring https://www.youtube.com/watch?v
Topic D: The Meaning of Even and Odd Numbers	Lesson 17	Relate doubles to even numbers and write number sentences to express the sums. https://www.youtube.com/watch?v
	Lesson 18	Pair objects and skip-count to relate to even numbers https://www.youtube.com/watch?v
	Lesson 19	Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers https://www.youtube.com/watch?v
	Lesson 20	Use rectangular arrays to investigate odd and even numbers https://www.youtube.com/watch?v
End-Module Assessment Task		

NJSLS Standards:

Module 6: Foundations of Multiplication and Division

2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

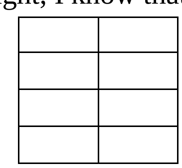
Second graders apply their work with doubles to the concept of odd and even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends or doubles addition facts (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Example: **Is 8 an even number? Justify your thinking.**

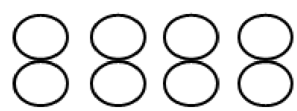
Student A
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number.

Student B
I grabbed 8 counters. I put them into 2 equal groups. There were 4 counters in each group, so 8 is an even number.

Student C
I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know that 8 is even.



Student D
I drew 8 circles. I matched one on the left with one on the right. Since they all match up I know that 8 is an even number.



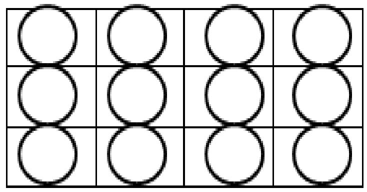
Student E
I know that 4 plus 4 equals 8. So 8 is an even number.

The focus of this standard is placed on the conceptual understanding of even and odd numbers. An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal parts. The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation and should not be used as the definition of an even number. (Van de Walle & Lovin, 2006, p. 292)

2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends

Second graders use rectangular arrays to work with repeated addition, a building block for multiplication in third grade. A rectangular array is any arrangement of things in rows and columns, such as a rectangle of square tiles. Students explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Due to the commutative property of multiplication, students can add either the rows or the columns and still arrive at the same solution.

Example: **What is the total number of circles below?**

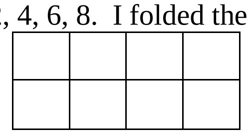


Student A	**Student B**
I see 3 counters in each column and there are 4 columns. So I added 3 + 3 + 3 + 3. That equals 12.	I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.
3 + 3 + 3 + 3 = 12	4 + 4 + 4 = 12

2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them

Second graders partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This work connects to the standard 2.OA.4 where students are arranging objects in an array of rows and columns. This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication.

Example:
Teacher: Partition the rectangle into 2 rows and 4 columns. How many small squares did you make?
Student: There are 8 squares in this rectangle. See- 2, 4, 6, 8. I folded the paper to make sure that they were all the same size.



M : Major Content S: Supporting Content A : Additional Content

Common addition and subtraction. [1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? **2 + 3 = ?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? **2 + ? = 5**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **? + 3 =5**
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?**5- 2 = ?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?**5 – ? = 3**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?**? -2 = 3**
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN** [2]
PUT TOGETHER / TAKE APART [3]	Three red apples and two green apples are on the table. How many apples are on the table? **3 + 2 = ?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? **3 + ? = 5, 5-3 = ?**	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? **5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1, 5 = 2 + 3, 5 = 3 + 2**
COMPARE	**DIFFERENCE UNKNOWN**	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? **2 + ? = 5, 5 – 2 = ?**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? **2 + 3 = ?, 3 + 2 = ?**	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **5 – 3 = ?, ? + 3 = 5**

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

[3] Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

[4] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/

Teaching Representations/ Manipulatives/ Tools:	
• Counters • Number bond • Number path	• Squares • Personal White board • Rectangular array

Terminology/ Symbols

- Array (an arrangement of objects in rows and columns)
- Columns (the vertical groups in a rectangular array)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Odd number (any number that is not even)
- Tessellation (tiling of a plane using one or more geometric shapes with no overlaps and no gaps)
- Whole number (e.g., 0, 1, 2, 3, …)
- Rows (the horizontal groups in a rectangular array)
- Repeated addition (e.g. 2+2+2)

Module 6 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math **Module 6:** ***Foundation of Multiplication and Division***			
Authentic Assessment: Playing Games	2.OA.3 2.OA.4	30 mins	Individual
Optional Mid-Module Assessment	2.OA.4	1 Block	Individual
Optional End-of-Module Assessment	2.OA.3 2.OA.4 2.G.2	1 Block	Individual

Second Grade Ideal Math Block

Getting Ready: Whole Group — 7-10 min.
Number Talk/ Number Strings

50-60 min.:

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Independent
Exit Ticket

CENTERS/STATIONS: Pairs / Small Group/ Individual — 15-20 min.
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

M:	A:	T:	H:
Meet with the teacher	**Application/ Problem Solving**	**Technology**	**Hands on Activities**

SUMMARY: Whole Group — Up to 5 min.
Lesson Closure: Student Reflection; Real Life Connections to Concept

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

Number Talks Cheat Sheet

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

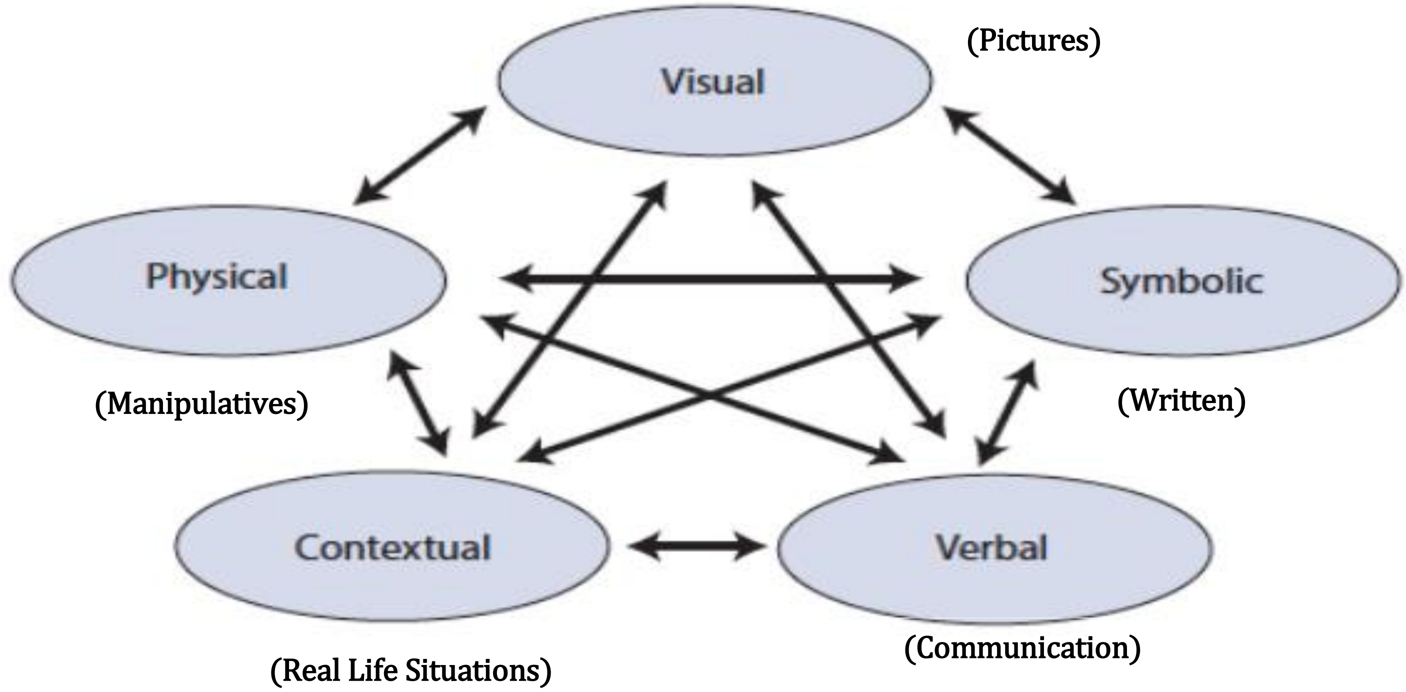
The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?
- What are Number Talks and Why are they

Student Name: ______________________________ Task: _____________________ School: ________ Teacher: __________ Date: ________

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	…a start 1	…getting there 2	…that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.
DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?
WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

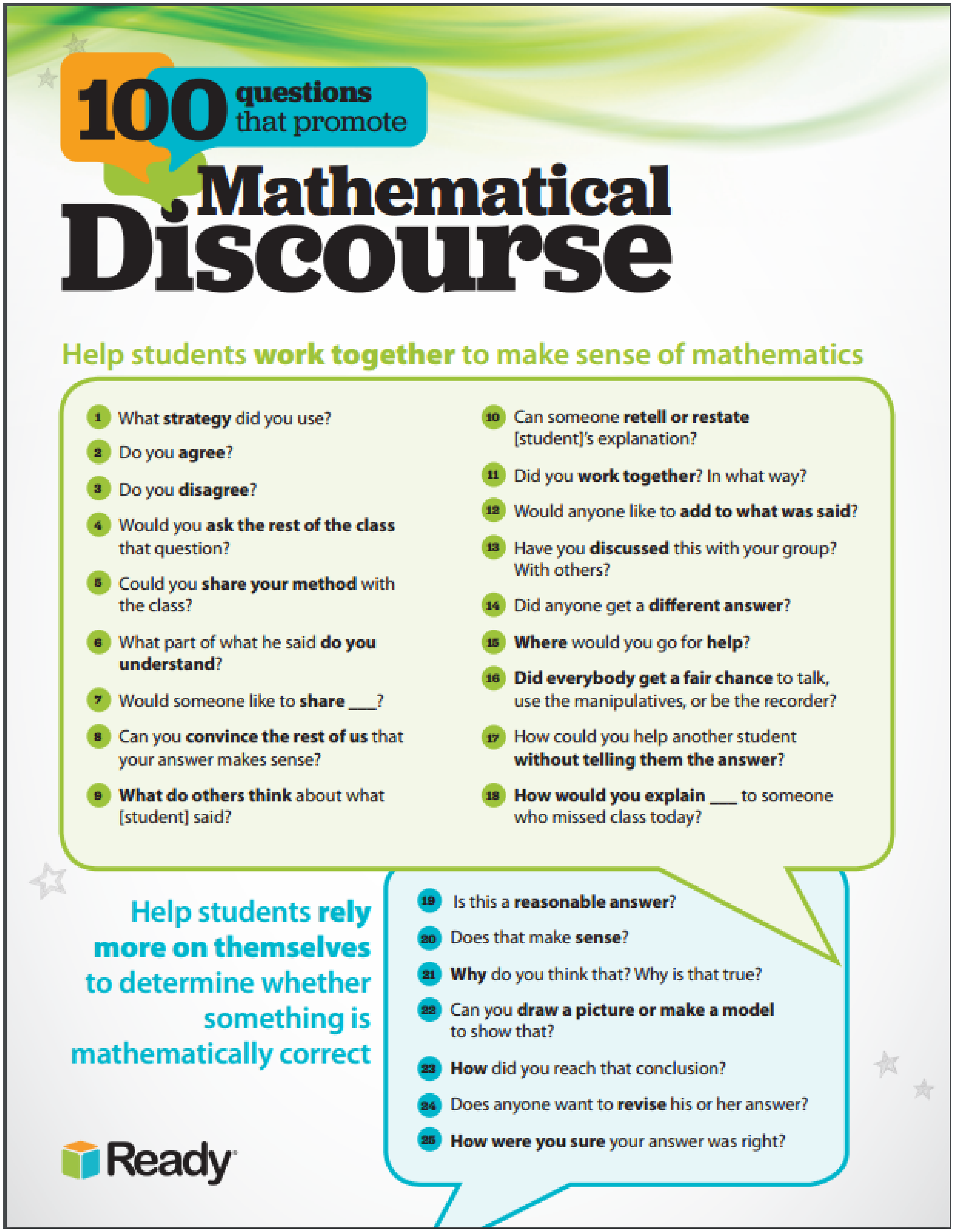
Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

<u>Teacher Questioning:</u>

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



Help students learn to **reason mathematically**

26. **How did you begin** to think about this problem?
27. What is **another way** you could solve this problem?
28. How could you **prove** ______?
29. Can you **explain how your answer is different from or the same as** [student]'s answer?
30. Let's **break the problem into parts**. What would the parts be?
31. Can you **explain this part more specifically**?
32. Does that **always work**?
33. Can you think of a case where that **wouldn't work**?
34. How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35. What do you need to do **next**?
36. What have you **accomplished**?
37. What are your **strengths and weaknesses**?
38. Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39. What is this problem about? What can you **tell me about it**?
40. Do you need to **define or set limits** for the problem?
41. How would you **interpret** that?
42. Could you **reword that in simpler terms**?
43. Is there something that can be **eliminated** or that is **missing**?
44. Could you **explain** what the problem is asking?
45. What **assumptions** do you have to make?
46. What do you **know** about this part?
47. Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.

1.OA.6 Add and Subtract within 10.

2.OA.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

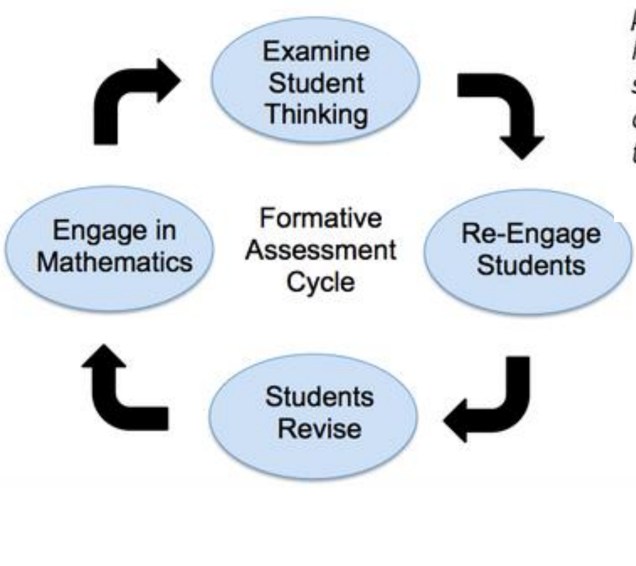
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	
1	**Make sense of problems and persevere in solving them**
	Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, “Does this make sense?” First Grade students’ conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences..
2	**Reason abstractly and quantitatively**
	Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: “There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?” first grade students may write 20 + 40 = 60 to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing 20 + 10 + 10 + 10 + 10 = 60. The numbers and equations written illustrate the students’ thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.

	Construct viable arguments and critique the reasoning of others
3	Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense.
	Model with mathematics
4	Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed.
	Use appropriate tools strategically
5	Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and higher-order thinking skills are developed

6	**Attend to precision** Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (=, >, 3, a proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger."
7	**Look for and make use of structure** Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation)
8	**Look for and express regularity in repeated reasoning** Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving 8+7+2, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the

	task “There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?” First Grade students realize that the 12 crayons could include 6 of each color (6+6 = 12), 7 of one color and 5 of another (7+5 = 12), etc. In essence, students repeatedly find numbers that add up to 12.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: ___________

NJSLS.:

____

____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE

Week of: ________________

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Second Grade PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		**Not There Yet** Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% **Distinguished command**	**PLD Level 4: 89%** **Strong Command**	**PLD Level 3: 79%** **Moderate Command**	**PLD Level 2: 69%** **Partial Command**	**PLD Level 1: 59%** **Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

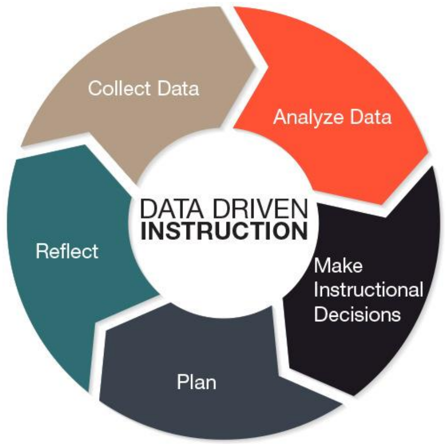
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form **School:** ________________ **Teacher:** ________________________ **Date:** _____________

Assessment: ____ **NJSLS:** ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

\

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

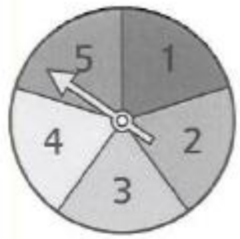
GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

A game company wants to create a new game. They are trying to figure out some of the rules of the game.

Part A
They want to use a spinner to distribute red and blue coins equally. If the spinner lands on an even number, the player gets a red coin. If the spinner lands on an odd number, the player gets a blue coin. If the game uses the spinner shown, how many numbers will result in a red coin? How many will result in a blue coin? Explain.



Part B
Will the spinner in Part A give the same number of red and blue coins? Explain.

Part C
Suppose gray coins are worth 3 points and black coins are worth 2 points. Write and solve an equation that shows how many points the gray coins are worth. Write and solve another equation that shows how many points the black coins are worth. Then write how many total points the player has.

Authentic Assessment: Playing Games Rubric

Part A	
Score	**Description**
2	Student response includes the following 2 elements • **Number of Red/Blue Coins:** 1 point • **Explanation:** 1 point <u>Sample Student Response:</u> 2 numbers will result in an even number and a red coin: 2, 4 3 numbers will result in an odd number and a blue coin: 1, 3, 5
1	Student response includes 1 of the 2 elements.
0	Student reponse is incorrect or irrelevant.

Part B	
Score	**Description**
2	Student response includes the following 2 elements • **Yes/No:** 1 point • **Explanation:** 1 point <u>Sample Student Response:</u> No, since there are 3 options for blue coins and 2 options for red coins, there will be more options for blue coins to be given than red coins.
1	Student response includes 1 of the 2 elements.
0	Student reponse is incorrect or irrelevant.

Part C	
Score	**Description**
3	Student response includes the following 3 elements • **Gray Coins:** 1 point • **Black Coins:** 1 point • **Total:** 1 point <u>Sample Student Response:</u> Gray: 3 + 3 = 6 points Black: 2 + 2 + 2 + 2 = 8 points Total: 6 + 8 = 14 points
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 2 elements.
0	Student reponse is incorrect or irrelevant.

Resources

Number Book Assessment Link: **http://investigations.terc.edu/**

Model Curriculum- http://www.nj.gov/education/modelcurriculum/

Georgia Department of Education: Games to be played at centers with a partner or small group. http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten

Engage NY: *For additional resources to be used during centers or homework.
https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf

Add/ Subtract Situation Types: Darker Shading indicates Kindergarten expectations
https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf

Math in Focus PD Videos: https://www-k6.thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.html

Number Talk/Strings: https://elementarynumbertalks.wordpress.com/second-grade-number-talks/

Suggested Literature

Fish Eyes by, Lois Ehlert

Ten Little Puppies by, Elena Vazquez

Zin! Zin! Zin! A Violin! by, Lloyd Moss

My Granny Went to the Market by, Stella Blackstone and Christopher Corr

Anno's Couting Book by, Mitsumasa Anno

Chicka, Chicka, 1,2,3 by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

How Dinosaurs Count to 10 by Jane Yolen and Mark Teague

10 Little Rubber Ducks by Eric Carle

Ten Black Dots by Donald Crews

Mouse Count by Ellen Stoll Walsh

Count! by Denise Fleming

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>